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Persistent Real Exchange Rates¹

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Abstract

Three well known facts that characterize exchange rate data are: (a) the high correlation between bilateral nominal and real exchange rates; (b) the high degree of persistence in real exchange rate movements; and (c) the high volatility of real exchange rates. This paper attempts a joint, albeit partial, rationalization of these facts in an environment with no staggered contracts and where prices are preset for only one quarter. There are two key innovations in the paper. First, we augment a standard two-country open economy model with learning-by-doing in production at the firm level. This induces monopolistically competitive firms to endogenize the productivity effect of their price setting behavior. Specifically, firms endogenously choose not to adjust prices by the full proportion of a positive monetary shock in order to take advantage of the productivity benefits of higher production. Second, we introduce habits in leisure. This makes the labor supply decision dynamic and adds an additional source of propagation. We show that the calibrated model can quantitatively reproduce significant fractions of the aforementioned facts. Moreover, as in the data, the model also produces a positive correlation between the terms of trade and the nominal exchange rate.

KEYWORDS: Real exchange rate movements, endogenous price stickiness, learning-by-doing

JEL CLASSIFICATION: F1, F2

1 Introduction

Three well known facts that characterize exchange rate data are: (a) the high correlation between bilateral nominal and real exchange rates; (b) the high degree of persistence in real exchange rate movements; and (c) the high volatility of real exchange rates. These facts have proved to be non-trivial challenges for standard open economy dynamic general equilibrium models. The conventional approach to explaining these facts has been to assume sticky prices and/or staggered contracts. However, recent work by Chari, Kehoe and McGrattan (2002) has called this approach into question. They show that in order for sticky prices and staggered contracts to account for a significant fraction of the dynamics of exchange rates, prices must be fixed for at least one year. Recent evidence in Bils and Klenow (2004) seriously calls into question such long-lived price stickiness.¹

In this paper we modify a standard two-country open economy model along two dimensions. First, we follow the work of Cooper and Johri (2002) and introduce a firm-level learning-by-doing effect into the production technology. In particular, higher production in any period by a firm leads to the accumulation of organizational capital by the firm. This causes increases not only in productivity in the next period but also in the stock of organizational capital in future periods. This feature makes the pricing decision of monopolistically competitive firms dynamic with firms endogeneizing the effect of their pricing decision today on productivity tomorrow. Hence, faced with a nominal shock firms voluntarily choose not to adjust their prices fully even when they are free to do so. This generates endogenous price stickiness thereby lowering the degree of exogenous stickiness that is required to match the data. Second, we introduce habit persistence in leisure. By making the household's labor supply decision dynamic, this feature generates an additional mechanism for the endogenous propagation of shocks.

In an environment where prices are preset for just one quarter and there is no staggered price

¹Bils and Klenow (2004) find that half of all posted prices last less than 4.3 months. Moreover, relative to the predictions of the standard sticky price models, they find that actual inflation displays far more volatility and less persistence even for goods which display high price stickiness.

setting, we quantitatively evaluate the response of the model economy to estimated money shocks. We find that the net impact of our two key innovations on the dynamics of real exchange rates is substantial. In their absence, real exchange rates would display essentially no auto-correlation. For our baseline parameterization, the first-order autocorrelation coefficient of the simulated real exchange rate series is 0.80 while the standard deviation of the real exchange rate relative to output is 5.67. The data counterparts of these two numbers are 0.94 and 5.5. The model can also quantitatively reproduce the observed correlation between the nominal and real exchange rates.² Lastly, the model produces a positive, contemporaneous correlation of 0.24 between the nominal exchange rate and the terms of trade which is consistent with data. As pointed out by Obstfeld and Rogoff (2000), most of the standard sticky price models are unable to produce this positive correlation.

We find these results interesting on two counts. First, our results are supportive of the two key modifications that we introduced in this paper – learning-by-doing and habits – for understanding real exchange rate movements. Second, we believe that our results demonstrate that long-lived sticky prices are not necessary to explain persistent real exchange rate movements since all our results are derived in an environment with only one-period preset prices.

While most of the elements in the model are standard, the idea behind the introduction of the two main additional features – learning-by-doing (LBD) and habit persistence in leisure – require a little elaboration. Consider a one-time permanent increase in money supply in the standard model without LBD and habit persistence. The standard model has the property that a nominal shock raises nominal demand for goods. If price-setting monopolistically competitive firms were free to reset their prices then they would raise their price to the point that real demand for their goods remains at the profit maximizing level of output. Hence, persistence of a nominal shock on real variables is linked to the length for which firms are unable to adjust prices.

How does the introduction of LBD alter the logic above? LBD makes the pricing decision of the

²We follow Chari, Kehoe and McGrattan (2002) and use their data on the USA and a European aggregate entity as the two countries for constructing the relevant moments in the data for our two country model.

firm dynamic. At each point in time, a firm that is choosing prices trades off the positive revenue effect of a higher price with the negative future productivity effect of lower learning today due to lower production. Hence, the optimal price, *ceteris paribus*, is lower than in the standard model. Moreover, this effect introduces an endogenous source of propagation of shocks. A higher output today implies a greater stock of organizational knowledge tomorrow (due to learning). This directly reduces the marginal cost of production tomorrow and hence, induces the firm to raise prices by less than they would otherwise. Crucially, this effect always operates independent of whether the learning effect is internal or external to the firm.

If the LBD effect is endogeneized by the firm then there is a second reason for sluggish price adjustment. In particular, a higher output today raises the stock of organizational capital tomorrow. Current output and current organizational capital stock are complementary inputs in producing future organizational capital. Hence, in the period after the shock when the firm is free to reset prices, it realizes that the higher current stock of organizational capital implies the marginal learning from an extra unit of output today is higher than in steady state. Put differently, learning is cheaper. At the margin, this induces the firm to raise prices less than it would otherwise in order to take advantage of the cheaper learning environment. Thus, a one-time permanent increase in money supply in a model with LBD would raise prices but only gradually toward its long run steady state level. The process eventually dies out due to decreasing returns to scale in the learning function. In the new steady state prices would have adjusted by the full proportion of the shock.

What role do habits play in propagating nominal shocks? A typical feature of models with sticky prices is that output is demand determined. Hence, when firms are unable to adjust their price they increase their production by primarily increasing labor employed. Once firms are free to adjust prices output declines back to its steady state level with the adjustment coming through a fall in employment. Habits in leisure make the labor supply decision of households dynamic. At an optimum the household balances the marginal utility gain of an extra unit of leisure today with not just the foregone wage but also the reduced marginal utility from an additional unit of leisure tomorrow due to the higher stock of habits. Hence, households adjust their labor supply slowly

over time which makes the output dynamics gradual in response to a shock. The flip side of this is that real wages adjust slowly which implies that prices which clear the goods market also adjust sluggishly relative to the standard model. This causes the real effects of nominal shocks to persist longer.

The intuition above suggests that the introduction of either learning or habits, by themselves, should increase the degree of persistence in the standard model. Indeed it does. Our quantitative results show that individually both LBD and habits raise the degree of persistence of real exchange rates relative to the standard model. However, neither effect is individually large enough to raise the persistence of real exchange rates close to the observed level in the data. When we introduce both effects simultaneously however, the persistence generated by the model rises significantly with the first-order autoregression coefficient of the real exchange rate in the model being 0.80 which is quite close to the 0.94 coefficient in the data. LBD and habits contribute roughly equal amounts to the overall degree of persistence generated by the model. Thus, to generate the correct quantitative magnitudes for the real exchange rate dynamics we need both learning and habits to operate simultaneously.

Our work is related to a large body of existing work on explaining real exchange rate fluctuations using models with sticky prices and monetary shocks. This literature goes back to Dornbusch (1976) but its modern general equilibrium version starts with Obstfeld and Rogoff (1995). Our work is probably closest in spirit to the papers by Betts and Devereaux (2000) and Chari, Kehoe and McGrattan (2002), both of which study models with monopolistic competition but introduce local currency pricing by exporting firms.³

A related literature has focussed on explaining deviations from purchasing power parity by using "pricing to market" arguments. This view formalizes the idea that producers can price discriminate across markets. Hence, price variations across countries arise due to different degrees

³This is in contrast to the producer currency pricing environment studied in Obstfeld and Rogoff (1995) where there is 100 percent exchange rate pass-through into domestic prices as purchasing power parity holds at all times on traded goods.

of market power (see Dornbusch, 1987). To get real exchange rates to be volatile and persistent this set-up can be augmented with distribution costs for traded goods with the distribution sector using non-traded resources. Formalizations of these arguments can be found in Burstein, Eichenbaum and Rebelo (2005), Obstfeld and Rogoff (2000) and Ravn and Mazzenga (2004) amongst others. We abstract from these distribution cost margins but show how the LBD margin introduced in this paper plays a similar role. Our work is also related to Bergin and Feenstra (2001) and Bouakez (2005) who consider more general demand functions than the constant elasticity case which allows for variable markups. Our model also generates fluctuating markups but not due to different demand functions. instead markups fluctuate due to the endogenous productivity effects induced by learning.

We also build on existing work on learning-by-doing (LBD) models by different authors. While learning-by-doing is often associated with workers and modeled as the accumulation of human capital, a number of economists have argued that firms are also store-houses of knowledge. Atkeson and Kehoe (2005) note “At least as far back as Marshall (1930, bk.iv, chap. 13.I), economists have argued that organizations store and accumulate knowledge that affects their technology of production. This accumulated knowledge is a type of unmeasured capital distinct from the concepts of physical or human capital in the standard growth model." Similarly Lev and Radharkrishnan (2003) write, “Organization capital is thus an agglomeration of technologies—business practices, processes and designs, including incentive and compensation systems—that enable some firms to consistently extract out of a given level of resources a higher level of product and at lower cost than other firms.”⁴

Our specification of how learning-by-doing leads to productivity increases draws on early work

⁴There are at least two ways to think about what constitutes organizational capital. Some, like Rosen (1972), think of it as a firm specific capital good while others focus on specific knowledge embodied in the matches between workers and tasks within the firm. While these differences are important, especially when trying to measure the payments associated with various inputs, they are not crucial to the issues at hand. As a result we do not distinguish between the two.

by Arrow (1962) and Rosen (1972) as well as a large empirical literature dating back roughly a hundred years. This literature documents the pervasive presence of learning effects in virtually every area of the economy. Recent studies include Bahk and Gort (1993), Irwin and Klenow (1994), Jarmin (1994), Benkard (2000), Thornton and Thompson (2001), Chang, Gomes and Schorfheide (2002) and Cooper and Johri (2002). The current specification is taken from Cooper and Johri (2002) which not only offers a detailed justification for the modeling assumptions but also a number of estimates of the learning technology at different levels of aggregation for the US economy.

The literature on habits in leisure goes back to Kydland and Prescott (1982). This body of work was motivated by the insight that in order to explain the volatility of labor hours time non-separability in leisure may be key. Numerous papers since have tried to estimate the size of habits on leisure and/or used them to explain the dynamic behavior of hours worked. A non-exhaustive list of these contributions are Hotz et al (1988), Eichenbaum et al (1988), Yun (1996) and Bouakez and Kano (2006). Our paper builds on this body of research.

The next section presents and develops the model while in Section 3 we describe the solution method and the calibration of the model. Section 4 compares the simulation results with the data and offers sensitivity tests while the last section contains concluding remarks.

2 Model

We consider a standard two-country open economy model as in Chari, Kehoe and McGrattan (2002) (CKM henceforth). We call the countries *home* and *foreign*. Each country is characterized by households, final goods firms, and intermediate goods firms. Intermediate goods are traded across countries while the final good is a non-tradable which is both a consumption and capital good. We assume that the state space is finite so that in each period one of a finitely many events may occur. We denote the history of events through time t by $s^t = (s_0, s_1, \dots, s_t)$ with an associated probability $\pi(s^t)$. Note that s_i is a specific state which includes policy shocks. Throughout the paper we shall adopt the notational convention of denoting foreign country variables with an asterisk.

2.1 Households

We assume that asset markets are complete so that households can trade in state-contingent securities which span all possible states. Households in the home country can buy state contingent nominal bonds $B(s^{t+1})$ which pay one unit of the domestic currency in state s^{t+1} and zero otherwise. Foreign households' holdings of these nominal one period bonds are denominated by B^* . The price of these bonds are given by $Q(s^{t+1}|s^t)$. In any period t households face the budget constraint

$$\begin{aligned} & P(s^t)c(s^t) + M(s^t) - M(s^{t-1}) + \sum_{s^{t+1}} Q(s^{t+1}|s^t)B(s^{t+1}) + P(s^t)x(s^t) \\ & \leq P(s^t) [w(s^t)N(s^t) + R(s^t)K(s^{t-1})] + B(s^t) + \Pi(s^t) + T(s^t), \end{aligned} \quad (1)$$

where M denotes nominal money balances, P is the price level, w is the wage rate, N is labor supply, K is the capital stock, x is investment, R is the rental rate for capital, Π are profits of intermediate goods producers, and T are lump sum transfers. We assume that the economy faces quadratic costs of adjusting the capital stock. In particular, the capital stock evolves according to

$$K(s^t) - (1 - \delta)K(s^{t-1}) = x(s^t) - \frac{v}{2} \left(\frac{x(s^t)}{K(s^{t-1})} - \delta \right)^2 K(s^{t-1}) \quad (2)$$

We should note that these adjustment costs are a real resource cost for the economy. However, since investment just equals depreciation in steady state, the particular form for the adjustment cost function assumed here guarantees that no adjustment costs are incurred in steady state.

Households in both countries derive utility from consumption, leisure and real money balances. Thus, households in the home country choose c, l, B, x and M to maximize their lifetime utility given by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U \left(c(s^t), l(s^t) - bl(s^{t-1}), \frac{M(s^t)}{P(s^t)} \right), \quad (3)$$

where β denotes the discount factor. Note that the preference specification above allows for endogenous habit formation with the $b \geq 0$ being the parameter which determines the degree of habit persistence. These preferences reduce to the standard specification with no habits for $b = 0$.

Households are endowed with one unit of time which they can either use for leisure l or work N .

Hence, $N + l = 1$ at all dates and states. The first-order conditions for optimality are given by

$$\frac{U_l(s^t)}{U_c(s^t)} = w(s^t) + b \sum_{s^{t+1}} Q(s^{t+1} | s^t) \frac{P(s^{t+1})}{P(s^t)} \frac{U_l(s^{t+1})}{U_c(s^{t+1})} \quad (4)$$

$$\frac{U_m(s^t)}{P(s^t)} - \frac{U_c(s^t)}{P(s^t)} = -\beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t) \frac{U_c(s^{t+1})}{P(s^{t+1})} \quad (5)$$

$$Q(s^{t+1} | s^t) = \beta \pi(s^{t+1} | s^t) \frac{U_c(s^{t+1})}{U_c(s^t)} \frac{P(s^t)}{P(s^{t+1})} \quad (6)$$

$$\frac{U_c(s^t)}{1 - v \left(\frac{x(s^t)}{K(s^{t-1})} - \delta \right)} = \beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t) \left[\frac{U_c(s^{t+1})}{1 - v \left(\frac{x(s^{t+1})}{K(s^t)} - \delta \right)} \right] D(s^{t+1}) \quad (7)$$

where $U_j(s^i)$ denotes the derivative of U with respect to variable j evaluated in state s^i . Note that

$$D(s^{t+1}) = R(s^{t+1}) \left[1 - v \left(\frac{x(s^{t+1})}{K(s^t)} - \delta \right) \right] + 1 - \delta + \frac{v}{2} \left[\left(\frac{x(s^{t+1})}{K(s^t)} \right)^2 - \delta^2 \right]$$

These first order conditions are standard. Equation (4) is the optimal labor-leisure choice. It is standard except for the second term on the right hand side which reflects the role of habits. In particular, an extra unit of leisure generates some positive current marginal utility but it also raises the stock of habits tomorrow. This second effect reduces the marginal utility of leisure tomorrow and hence is an additional cost (over and above the foregone wage) to current leisure. (5) is the optimality condition determining money demand while equation (6) is the equation which determines optimal bond holdings. Lastly, equation (7) is the optimality condition for capital accumulation. This condition looks slightly different from the standard intertemporal euler equation due to the presence of adjustment costs. Since $v \geq 0$, at the optimum, the household endogenizes the fact that one unit of foregone consumption today produces only $1 - v \left(\frac{x(s^t)}{K(s^{t-1})} - \delta \right)$ units of capital tomorrow. Also, note that $\pi(s^{t+1} | s^t) = \pi(s^{t+1})/\pi(s^t)$ is the probability of state s^{t+1} conditional on state s^t having been realized.

The preceding set of first order conditions imply two no-arbitrage conditions:

$$\frac{U_m(s^t)}{U_c(s^t)} = 1 - \sum_{s^{t+1}} Q(s^{t+1} | s^t)$$

$$\frac{P(s^t)}{1 - v \left(\frac{x(s^t)}{K(s^{t-1})} - \delta \right)} = \sum_{s^{t+1}} Q(s^{t+1} | s^t) \frac{P(s^{t+1})D(s^{t+1})}{1 - v \left(\frac{x(s^{t+1})}{K(s^t)} - \delta \right)}$$

The first is the no-arbitrage relationship between saving in money balances and in nominal state contingent bonds. Note that $\sum_{s^{t+1}} Q(s^{t+1} | s^t)$ is the total expenditure on nominal bonds that is required for a guaranteed delivery of one unit of the domestic currency in state s^{t+1} . Hence, this defines the nominal interest rate. The second equation is the no-arbitrage relation between bonds and capital. $P(s^t)$ is the nominal cost of one unit of foregone consumption which produces $1 - v \left(\frac{x(s^t)}{K(s^{t-1})} - \delta \right)$ units of capital. This accumulated capital delivers $P(s^{t+1})D(s^{t+1}) / \left[1 - v \left(\frac{x(s^{t+1})}{K(s^t)} - \delta \right) \right]$ units of the domestic currency in s^{t+1} since the accumulated capital today also reduces the adjustment cost required for capital accumulation tomorrow. The purchase price of a bond which delivers one unit of the local currency in state s^{t+1} is $Q(s^{t+1} | s^t)$. The right hand side of the second equation thus gives the future benefit of foregoing one unit of consumption today evaluated through bond prices.

The foreign households face a symmetric problem to the domestic household. Their periodic budget constraint is given by

$$P^*(s^t)c^*(s^t) + M^*(s^t) - M^*(s^{t-1}) + \sum_{s^{t+1}} Q(s^{t+1} | s^t) \frac{B^*(s^{t+1})}{e(s^t)} + P^*(s^t)x^*(s^t) \quad (8)$$

$$\leq P^*(s^t) [w^*(s^t)l^*(s^t) + R^*(s^t)K^*(s^{t-1})] + \frac{B^*(s^t)}{e(s^t)} + \Pi^*(s^t) + T^*(s^t),$$

where e denotes the domestic currency price of one unit of the foreign currency, i.e., it is the nominal exchange rate. Moreover, capital accumulation in the foreign country is given by

$$K^*(s^t) - (1 - \delta) K^*(s^{t-1}) = x^*(s^t) - \frac{v}{2} \left(\frac{x^*(s^t)}{K^*(s^{t-1})} - \delta \right)^2 K^*(s^{t-1}). \quad (9)$$

Note that we are assuming that the technology governing adjustment costs is identical in both countries, i.e., they both face the same adjustment cost parameter v .

This problem leads to three analogous first order conditions. The first order conditions for the labor-leisure, money balances and capital accumulation are exactly symmetric to the domestic household. The optimal bond holdings equation for the foreign household is given by

$$Q(s^{t+1}|s^t) = \beta \pi(s^{t+1}|s^t) \frac{U_c^*(s^{t+1})}{U_c^*(s^t)} \frac{P^*(s^t)e(s^t)}{P^*(s^{t+1})e(s^{t+1})}$$

Equating the state contingent bond prices for home and foreign households gives

$$\frac{q(s^{t+1})}{q(s^t)} = \frac{U_c^*(s^{t+1})/U_c(s^{t+1})}{U_c^*(s^t)/U_c(s^t)}.$$

where we have defined the real exchange rate as $q = \frac{eP^*}{P}$. Iterating on this equation yields the expression

$$q(s^t) = \kappa \frac{U_c^*(s^t)}{U_c(s^t)}, \quad (10)$$

where $\kappa = \frac{U_c(s^0)}{U_c^*(s^0)} \frac{e(s^0)P^*(s^0)}{P(s^0)}$. Equation (10) makes clear that the real exchange rate in this economy is just given by the ratio of marginal utilities between home and abroad.

Due to the complete asset markets environment our model generates the consumption correlation puzzle. As is obvious from equation (10) the correlation between relative consumption and the real exchange rate is going to be too high (it equals unity in the case where preferences are separable across the three arguments) when compared with the data. This is a standard problem in models with complete asset markets and our's is no exception. Recent work by Corsetti et al (2006) who introduce distribution costs and Kocherlakota and Pistaferri (2006) who relax the assumption of perfect risk sharing within countries have both tried to address this issue.

2.2 Final goods firms

In both countries final goods are produced using a continuum of domestic and foreign intermediate goods which are indexed by $i \in [0, 1]$. The production technologies for the final goods sector are given by

$$y(s^t) = \left[a_1 \left(\int_0^1 y_h(i, s^t)^\theta di \right)^{\frac{\rho}{\theta}} + a_2 \left(\int_0^1 y_f(i, s^t)^\theta di \right)^{\frac{\rho}{\theta}} \right]^{1/\rho}, \quad (11)$$

$$y^*(s^t) = \left[a_1 \left(\int_0^1 y_f^*(i, s^t)^\theta di \right)^{\frac{\rho}{\theta}} + a_2 \left(\int_0^1 y_h^*(i, s^t)^\theta di \right)^{\frac{\rho}{\theta}} \right]^{1/\rho}. \quad (12)$$

Thus, the elasticity of substitution between home and foreign intermediate goods is $1/(1-\rho)$ while the elasticity of substitution between intermediate goods produced in the same location is $1/(1-\theta)$.⁵

The final goods sector is assumed to be competitive in both countries. Thus, final goods firms in the home country choose $y_h(i, s^t)$ and $y_f(i, s^t)$ to maximize

$$P(s^t)y(s^t) - \int_0^1 P_h(i, s^{t-1})y_h(i, s^t)di - \int_0^1 P_f(i, s^{t-1})y_f(i, s^t)di$$

subject to equation (11). $P_h(i, s^{t-1})$ is the price of the home intermediate good i while $P_f(i, s^{t-1})$ is home currency price of the foreign intermediate good i . Note that these expressions reflect the fact that intermediate goods producers set prices for period t before observing s^t . Hence, intermediate goods prices are preset one period in advance. Also, intermediate goods producers set their prices in the currency of the country where they sell. This set up assumes local currency pricing as well as pricing to market (see Betts and Devereaux (2000)).

The solution to this problem leads to the following input demand functions:

$$y_h(i, s^t) = (a_1 P(s^t))^{\frac{1}{1-\rho}} \frac{\bar{P}_h(s^{t-1})^{\frac{\rho-\theta}{(1-\rho)(\theta-1)}}}{P_h(i, s^{t-1})^{1/(1-\theta)}} y(s^t), \quad (13)$$

$$y_f(i, s^t) = (a_2 P(s^t))^{\frac{1}{1-\rho}} \frac{\bar{P}_f(s^{t-1})^{\frac{\rho-\theta}{(1-\rho)(\theta-1)}}}{P_f(i, s^{t-1})^{1/(1-\theta)}} y(s^t), \quad (14)$$

where $\bar{P}_j(s^{t-1}) = \left(\int_0^1 (P_j(i, s^{t-1}))^{\frac{\theta}{\theta-1}} di \right)^{\frac{\theta-1}{\theta}}$ for $j = h, f$. Moreover, the zero profit condition for final goods firms implies that the price of the final good in the home country is given by

$$P(s^t) = \left(a_1^{\frac{1}{1-\rho}} \bar{P}_h(s^{t-1})^{\frac{\rho}{\rho-1}} + a_2^{\frac{1}{1-\rho}} \bar{P}_f(s^{t-1})^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}}. \quad (15)$$

An analogous problem for foreign final goods producers implies two additional input demand equations and an equation determining the foreign currency price of the final good in the foreign

⁵Our notational convention for intermediate goods is that the subscript denotes origin of the good while the superscript denotes the destination of the good. Hence, y_h^* denotes the home intermediate good sold in the foreign country while y_h denotes the home intermediate good sold at home. The foreign intermediates follow similarly.

country:

$$y_f^*(i, s^t) = (a_1 P^*(s^t))^{\frac{1}{1-\rho}} \frac{\bar{P}_f^*(s^{t-1})^{\frac{\rho-\theta}{(1-\rho)(\theta-1)}}}{P_f^*(i, s^{t-1})^{1/(1-\theta)}} y^*(s^t) \quad (16)$$

$$y_h^*(i, s^t) = (a_2 P^*(s^t))^{\frac{1}{1-\rho}} \frac{\bar{P}_h^*(s^{t-1})^{\frac{\rho-\theta}{(1-\rho)(\theta-1)}}}{P_h^*(i, s^{t-1})^{1/(1-\theta)}} y^*(s^t) \quad (17)$$

$$P^*(s^t) = \left(a_1^{\frac{1}{1-\rho}} \bar{P}_f^*(s^{t-1})^{\frac{\rho}{\rho-1}} + a_2^{\frac{1}{1-\rho}} \bar{P}_h^*(s^{t-1})^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}} \quad (18)$$

2.3 Intermediate goods firms

As is standard in this class of models, monopolistically competitive firms solve a two stage problem. In stage 1 they choose their factor inputs optimally to produce a given output of the good. In stage 2, they choose prices to maximize profits subject to the demand functions they face for their product taking as given the optimal cost function derived in stage 1. Home intermediate goods firms produce goods for both the home final goods sector as well as for the foreign final goods sector.

The production technology facing intermediate goods firm i at home is

$$y(i, s^t) \equiv y_h(i, s^t) + y_f^*(i, s^t) = F(K(i, s^{t-1}), N(i, s^t), H(i, s^{t-1})),$$

where H is the stock of organizational capital of the firm. More specifically, we shall assume that the production technology is given by

$$F(K(i, s^{t-1}), N(i, s^t), H(i, s^{t-1})) = N(i, s^t)^\alpha K(i, s^{t-1})^\phi H(i, s^{t-1})^\varepsilon$$

Capital and labor are rented in competitive factor markets. The presence of organizational capital in the production technology of intermediate goods is one of the two key innovations in this paper relative to CKM (2002). We follow Cooper and Johri (2002) and assume that the evolution of firm specific organizational capital is given by

$$H(i, s^t) = H(i, s^{t-1})^\gamma y(i, s^t)^\eta. \quad (19)$$

Thus, an intermediate goods firm's output depends positively on its output yesterday due to the link through the stock of organizational capital.

2.3.1 Stage I problem: Cost minimization

In Stage I domestic intermediate goods firm i chooses labor and capital inputs to minimize cost given a level of demand. Thus, this firm minimizes

$$C(i, s^t) = P(s^t) [w(s^t)N(i, s^t) + R(s^t)K(i, s^{t-1})]$$

subject to $N(i, s^t)^\alpha K(i, s^{t-1})^\phi H(i, s^t)^\varepsilon \geq y(i, s^t)$. The first order condition for this problem is the familiar relation

$$\frac{w(s^t)}{R(s^t)} = \frac{\alpha}{\phi} \frac{K(i, s^{t-1})}{N(i, s^t)} \quad \text{for all } i$$

Thus, the optimal capital-labor ratio is identical across all firms. Substituting this back into the cost function gives the optimized costs to be

$$\tilde{C}(i, s^t) = P(s^t)B \left(w(s^t)^\alpha R(s^t)^\phi \frac{y(i, s^t)}{H(i, s^{t-1})^\varepsilon} \right)^{\frac{1}{\alpha+\phi}} \quad (20)$$

where $B = \left(\frac{\alpha+\phi}{\phi} \right) \left(\frac{\phi}{\alpha} \right)^{\frac{\alpha}{\alpha+\phi}}$. In deriving \tilde{C} we have also used the production function for intermediate goods. Note that when $\alpha + \phi = 1$, the minimized cost function is linear in output. For later reference, it is useful at this stage to note that \tilde{C} is decreasing in H . Hence, a bigger stock of organizational capital reduces operating costs for the firm. Lastly, an analogous expression holds for foreign intermediate goods firms, i.e.,

$$\tilde{C}^*(i, s^t) = P^*(s^t)B \left(w^*(s^t)^\alpha R^*(s^t)^\phi \frac{y^*(i, s^t)}{H^*(i, s^{t-1})^\varepsilon} \right)^{\frac{1}{\alpha+\phi}} \quad (21)$$

where $y^*(i, s^t) \equiv y_f(i, s^t) + y_f^*(i, s^t)$.

2.3.2 Stage II problem

In stage II the intermediate goods firms jointly choose organizational capital for tomorrow, $H(i, s^t)$ and the nominal prices that they post for the period, P_h and P_h^* . Note that these prices are in the local currency of the market where they are selling. Crucially, at time t firms post these prices before observing the monetary shocks for this period. Thus, the posted prices for time t are based only on information contained in the history s^{t-1} .

Each domestic firm maximizes the present discounted value of profits given by

$$\sum_t \sum_{s^t} Q(s^t) \Pi(i, s^t) = \sum_t \sum_{s^t} Q(s^t) \left[P_h(i, s^{t-1}) y_h(i, s^t) + e(s^t) P_h^*(i, s^{t-1}) y_h^*(i, s^t) - \tilde{C}(i, s^t) \right].$$

The maximization is subject to the constraints imposed by equations (13), (17), (19) and (20). Note that $Q(s^t) = Q(s^{t-1})Q(s^t | s^{t-1})$. The first order conditions for firm optimality once suitably rearranged yield

$$P_h(i, s^{t-1}) = \frac{\sum_{s^t} Q(s^t) y_h(i, s^t) \{mc(i, s^t) - \eta \lambda(i, s^t) H(i, s^{t-1})^\gamma y(i, s^t)^{\eta-1}\}}{\theta \sum_{s^t} Q(s^t) y_h(i, s^t)}, \quad (22)$$

$$P_h^*(i, s^{t-1}) = \frac{\sum_{s^t} Q(s^t) y_h^*(i, s^t) \{mc(i, s^t) - \eta \lambda(i, s^t) H(i, s^{t-1})^\gamma y(i, s^t)^{\eta-1}\}}{\theta \sum_{s^t} Q(s^t) e(s^t) y_h^*(i, s^t)}, \quad (23)$$

$$\lambda(i, s^t) = \sum_{s^{t+1}} Q(s^{t+1}) \left\{ \left(\frac{\varepsilon}{\alpha + \theta} \right) \frac{\tilde{C}(i, s^{t+1})}{H(i, s^t)} + \gamma \lambda(i, s^{t+1}) H(i, s^t)^{\gamma-1} y(i, s^{t+1})^\eta \right\}, \quad (24)$$

where $mc = \frac{\partial \tilde{C}}{\partial y}$ is the marginal cost of producing an extra unit of output. Note that λ is the multiplier associated with equation (19).

The first two equations (22 and 23) give the optimal prices set by domestic intermediate firms at home and abroad respectively while the third equation determines the optimal accumulation of organizational capital. The pricing equations are standard except for the second term within curly brackets in the numerator. This term reflects the fact that the firm takes into account that its pricing decision today affects organizational capital tomorrow through the effect on demand and hence output. Equation (24) shows that the value of an additional unit of organizational capital reflects both its implied cost savings tomorrow as well as its positive effect on the future stock of organizational capital.

For future reference it is worth noting at this stage that if learning were external to the firm then the second term in the numerator of both equations (22 and 23) would be absent from the optimal pricing equation thereby making it look more standard. However, learning effects would still show up in the model through the marginal cost term. In particular, with exogenous learning, higher current output would raise organizational capital tomorrow thereby reducing the marginal cost tomorrow. This would induce lower prices tomorrow.

The problem for the intermediate goods firms abroad leads to a symmetric set of optimality conditions. In particular, we have

$$\begin{aligned}
P_f^*(i, s^{t-1}) &= \frac{\sum_{s^t} Q(s^t) y_f^*(i, s^t) \{e(s^t) m c^*(i, s^t) - \eta \lambda^*(i, s^t) H^*(i, s^{t-1}) \gamma y^*(i, s^t)^{\eta-1}\}}{\theta \sum_{s^t} Q(s^t) e(s^t) y_f^*(i, s^t)}, \\
P_f(i, s^{t-1}) &= \frac{\sum_{s^t} Q(s^t) y_f(i, s^t) \{e(s^t) m c^*(i, s^t) - \eta \lambda^*(i, s^t) H^*(i, s^{t-1}) \gamma y^*(i, s^t)^{\eta-1}\}}{\theta \sum_{s^t} Q(s^t) y_f(i, s^t)}, \\
\lambda^*(i, s^t) &= \sum_{s^{t+1}} Q(s^{t+1}) \left\{ \left(\frac{\varepsilon}{\alpha + \phi} \right) \frac{e(s^t) \tilde{C}^*(i, s^{t+1})}{H^*(i, s^t)} + \gamma \lambda^*(i, s^{t+1}) H^*(i, s^t)^{\gamma-1} y^*(i, s^{t+1})^\eta \right\}.
\end{aligned}$$

These equations have the same intuitive explanations as the ones for the domestic firm. The only point worth noting is that since all the accounting is being done in terms of domestic currency, and since the foreign firms face their costs in foreign currency, the cost terms have to be multiplied by e to convert them into domestic currency units.

2.4 Government

We are going to consolidate the fiscal and monetary authorities in this environment into one entity called the government. The government injects money into the economy through lump-sum transfers. Hence,

$$T(s^t) = M(s^t) - M(s^{t-1}).$$

Moreover, we assume that the money supply process is given by

$$M(s^t) = \mu(s^t) M(s^{t-1}),$$

where $\mu(s^t)$ is a stochastic process and where $M(s^{-1})$ is given. We further assume that the foreign government behaves symmetrically so that analogous conditions hold for them.

2.5 Equilibrium conditions

For an equilibrium in this environment some market clearing conditions need to be satisfied. In particular, the final goods market has to clear in both countries. For the home country then,

$$c(s^t) + x(s^t) = y(s^t).$$

A symmetric condition must hold in the foreign country.

Labor and capital market clearing at home require that (a) $\int_0^1 l(i, s^t) di = l(s^t)$, and (b) $\int_0^1 K(i, s^t) di = K(s^{t-1})$. The key thing to note in the condition for capital market clearing is that the aggregate supply of physical capital in the economy at time t is determined in the previous period based on state s^{t-1} while the demand for capital by firms at time t reflects the state s^t . Symmetric conditions hold for the foreign country. Lastly, the bond market clearing condition dictates that total bonds in circulation in the world must be in zero net supply. Hence, $B + B^* = 0$.

Two features of this economy are worth noting. First, since all the home (foreign) intermediate goods enter symmetrically in the production function, in any symmetric equilibrium we must have $P_j(i, s^t) = P_j(s^t) = \bar{P}_j(s^t)$ for all i and for $j = h, f$. Second, the domestic market clearing condition combined with the consumers budget constraint and government transfers yields a standard current account equation for the home country:

$$\sum_{s_{t+1}} Q(s^{t+1} | s^t) B(s^{t+1}) = B(s^t) + e(s^t) P_h^*(s^{t-1}) y_h^*(s^t) - P_f(s^{t-1}) y_f(s^t)$$

Since $B = -B^*$, there is only one independent current account equation. The surplus of one country is the deficit of the other.

We conclude this section by defining the equilibrium for the economy:

Definition 1 *An equilibrium in this economy is a set of allocations for consumers $c(s^t)$, $c^*(s^t)$, $l(s^t)$, $l^*(s^t)$, $M(s^t)$, $M^*(s^t)$, $x(s^t)$, $x^*(s^t)$, $K(s^t)$, $K^*(s^t)$, $B(s^t)$, $B^*(s^t)$; allocations and prices for intermediate firms $y_h(i, s^t)$, $y_h^*(i, s^t)$, $y_f(i, s^t)$, $y_f^*(i, s^t)$, $N(i, s^t)$, $N^*(i, s^t)$, $K(i, s^t)$, $K^*(i, s^t)$, $P_h(i, s^{t-1})$, $P_h^*(i, s^{t-1})$, $P_f(i, s^{t-1})$, $P_f^*(i, s^{t-1})$ for all $i \in [0, 1]$; allocations and prices for final goods firms, $y(s^t)$, $y^*(s^t)$, $P(s^t)$, $P^*(s^t)$; real input prices $w(s^t)$, $w^*(s^t)$, $R(s^t)$, $R^*(s^t)$; and bond prices $Q(s^{t+1} | s^t)$ such that (a) consumers solve their optimization problem; (b) intermediate firms solve their profit maximization problem; (c) final goods firms solve their profit maximization problem; (d) all markets clear; and (e) the restrictions imposed by the government transfer policy are satisfied.*

3 Computation method and calibration

The model is solved using the method outlined in King and Watson (2002) using a linear approximation to the system of equations outlined above. We solve for the stationary equilibrium. Nominal variables that are growing in steady state are rendered stationary by dividing by the stock of money in the economy.

In order to simulate the economy, functional forms have to be specified and values assigned to a number of parameters. In order to offer a consistent comparison we have chosen these to be as close as possible to the benchmark specification in CKM (2002). Our specification of preferences is

$$U\left(c(s^t), l(s^t) - bl(s^{t-1}), \frac{M(s^t)}{P(s^t)}\right) = \frac{1}{1-\sigma} Z(s^t)^{1-\sigma} + \psi [l(s^t) - bl(s^{t-1})]^{1-\xi} / (1-\xi). \quad (25)$$

where

$$Z(s^t) = \left[\omega c(s^t)^{(\varphi-1)/\varphi} + (1-\omega) \left(\frac{M(s^t)}{P(s^t)} \right)^{(\varphi-1)/\varphi} \right]^{\frac{\varphi}{\varphi-1}}$$

Following CKM we set $\omega = 0.94$, $\varphi = 0.39$. They derive these estimates by taking logs of the first order condition for money balances and then running an OLS regression on the resulting expression for money demand. We pick σ to match the volatility of the real exchange rate relative to output in the model with the data.⁶ ψ is set so that the fraction of the time endowment spent on working in steady state is 0.3. In order to maintain consistency of preferences with balanced growth we set $\xi = \sigma$. In terms of the habit persistence parameter, a number of estimates of b are available in the literature ranging from a high of roughly 0.8 in Eichenbaum, Hansen and Singleton (1988) to a low around 0.5 in Braun and Evans (1998). Following Eichenbaum, Hansen and Singleton (1988) we set $b = 0.8$. Lastly, as in Cooper and Johri (2002) we set the discount factor β to 0.984.

The four parameters appearing in the final goods technology are ρ, θ, a_1 and a_2 . The parameter ρ governs the elasticity of substitution between domestic and foreign goods. $\rho = 1/3$ implies an elasticity of 1.5 which is the number used by Backus, Kehoe and Kydland (1994). We set $a_1 = 0.94$ and $a_2 = 0.06$. These numbers, from CKM (2002), were chosen to set the steady state share of

⁶We report the sensitivity of our results to σ below in our robustness checks.

imports in total US GDP to 1.6 percent (which is the share of US imports from Europe in the data). Note that in a symmetric steady state $y_h/y_f = (a_1/a_2)^{\frac{1}{1-\rho}}$.

For the parameters governing learning (ε, γ and η), we build on the vast number of empirical studies of learning-by-doing summarized in Irwin and Klenow (1994). There are a variety of available estimates on learning which vary from 20 percent learning (a doubling of production experience leads to a 20 percent fall in costs, often referred to as the "20 percent rule") to 39 percent learning estimated by Benkard (2000). These roughly correspond to $\varepsilon = .27$ and $\varepsilon = .48$ respectively. We choose $\varepsilon = 0.4$, a number in the middle of this range, corresponding to 31 percent learning. These studies assume that current production contributes fully to the stock of organizational capital. Following the literature we retain a value of $\eta = 1$. We set $\gamma = 0.5$ as estimated by Cooper and Johri (2002).

The steady state capital output ratio is a function of the technological parameters as well as β, δ and θ . We set the labor and capital share parameters to conventional levels: $\alpha = 0.6$ and $\phi = 0.4$. The depreciation rate, $\delta = 0.021$, was borrowed from CKM. We choose θ (which governs the elasticity of substitution between domestic intermediate goods) to maintain the steady state capital output ratio at 10.2. This is a quarterly number which converts to an annual capital output ratio of 2.1.⁷ The adjustment cost parameter is varied across specifications to keep the ratio of the standard deviation of investment to output at the value found in the US data. Table 1 lists all our parameter values for the baseline calibration of the model.

For the money supply process, we again follow CKM and postulate the following processes for home and abroad:

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + \varepsilon_{\mu t} \tag{26}$$

$$\log \mu_t^* = \rho_\mu \log \mu_{t-1}^* + \varepsilon_{\mu t}^*$$

⁷In the standard model the mark-up of price over marginal cost is given by $1/\theta$. Hence, the typical approach is to set $\theta = 0.9$ since this produces a price mark-up of 10 percent. In our model, the mark-up isn't a constant since it also depends on the learning effects. We should point out that the steady state mark-up induced by our baseline parameterization is 5 percent.

where $\varepsilon_{\mu t}$ distributed as $N(0, \sigma_\mu^2)$. The shocks are positively cross-correlated. CKM estimate ρ_μ by running this regression on US data for M1 from 1959:2 to 2001:1. We use their estimate for ρ_μ and set it to 0.68. The correlation between shocks in the two countries was chosen to match the cross-country correlation of output, as in CKM (2002).

4 Results

The main goal of this section is to evaluate the quantitative performance of this model. We are especially interested in the real exchange rate dynamics generated by the model. We will focus on the same two moments emphasized in the literature: the first order autocorrelation of the real exchange rate (ρ_q) and it's standard deviation relative to output ($\sigma_{q/y}$) where all variables are measured as percent deviations from their steady state values. The behaviour of hours, investment and consumption is an important part of our story so we will discuss the volatility of these as well relative to the volatility of output.

Table 2 summarizes our main results. All data (in logs) are linearly detrended before computing the reported statistics. Row 1 reports the relevant statistics for the US economy. Columns 2-6 of Table 2 present the volatility of consumption, investment, hours, real exchange rate, and nominal exchange rate relative to the volatility of output. Columns 7 and 8 report the first order autocorrelation coefficient of the nominal and real exchange rates for various models and the US economy relative to a European aggregate. The last column reports the cross-correlation between nominal and real exchange rates. We use the notation $\sigma_{i/j}$ to denote the standard deviation of variable i relative to the standard deviation of variable j . $\rho_{i,j}$ denotes the correlation between i and j .

In the data the real and nominal exchange rates are highly positively correlated, very persistent and highly volatile. Row 1 shows that the real exchange rate between US and Europe is roughly five times as volatile as US aggregate output while the nominal exchange rate is six times as volatile. Moreover, both the nominal and real exchange rates have a first order autocorrelation coefficient larger than 0.9.

Our model reduces to the standard model used in the literature with one-period pre-set prices when we shut down both the learning-by-doing (LBD) and habit persistence effects. We refer to this as the *benchmark model*. Row 2 of Table 2 reports the statistics generated by this benchmark model. The benchmark model with prices pre-set for one period is able to deliver approximately the same amount of relative volatility as the data (the underlying relative risk aversion parameter is 11). However, not surprisingly, it fails to deliver any persistence in the real exchange rate. The autocorrelation coefficient of q is negative and close to zero! The nominal exchange rate, e , inherits roughly the same persistence that is built into the money shock process. The benchmark model also displays too high a volatility of aggregate hours relative to output and too low a relative volatility of consumption.

The third row of Table 2 reports the statistics generated by the full model including both LBD and habits. Clearly, the model fits the data significantly better than the benchmark model. While most of the other statistics are comparable, our baseline model does much better in terms of the persistence of the real exchange rate. The first-order autocorrelation coefficient of the real exchange rate is 0.8 which is a major improvement relative to -0.01 in the benchmark model. Moreover, the relative volatility of the real exchange rate continues to be very close to the data for the same relative risk aversion parameter $\sigma = 11$. This isn't a surprise since in the complete markets case the variance of the real exchange rate is proportional to the intertemporal risk aversion parameter. Hence, as shown by CKM (2002), one can always increase the intertemporal risk aversion parameter sufficiently in order to reproduce the real exchange rate volatility in the data.

Row four reproduces moments using CKM's benchmark specification with prices preset for four quarters but absent our mechanisms.⁸ Even with long periods of sticky prices, the model generates real exchange rates that are somewhat less persistent and less volatile than those reported in row three. The relative volatilities of consumption and investment differ from other rows of Table 2 due to different calibration strategies which are discussed below. As we show in our sensitivity analysis, this leads to only small changes in exchange rate behaviour.

⁸Note that the moments are based on linear detrending to be compatible with the rest of the table.

Clearly the model including both LBD and habits does quite well in generating a quantitatively significant amount of persistence in the real exchange rate though a little less than seen in the data. But what is the contribution of each margin? Rows 5 and 6 of Table 1 answer this question. The results show that both learning-by-doing (LBD) and habits on their own improve upon the benchmark model. Once again both models are calibrated to deliver the same relative volatility of investment as seen in the data. Moreover, the risk aversion parameter, σ , is chosen to deliver roughly the same relative volatility of real and nominal exchange rates as before. As a result the two models look quite similar on these dimensions. However, the implications of the two models are starkly different when one considers the persistence of q . Compared to a value of -0.01 in the benchmark model, the model with only LBD delivers an autocorrelation coefficient of 0.48 while the model with only habits delivers a coefficient of 0.45. Hence, both margins significantly increase the degree of persistence of the real exchange rate.

A key feature of our model is that learning is internal to the firm. Hence, the firm takes learning into account while making its pricing decisions. This is precisely the feature that makes the pricing decision of the firm dynamic. However, if learning were exogenous to the firm but continued to follow an evolution process given by equation (19), there would still be a propagation mechanism built into the model through the evolution of H . Specifically, if output increased in any period then it would set in motion an increase in organizational capital H which would, in turn, increase output directly and indirectly through the transition mechanism of equation (19). Thus, a question of interest is how much of the persistence of q is being driven by this exogenous transmission mechanism and how much more is coming from the endogenous pricing decision of the firm?

The last row of Table 2 reports the statistics for the model with both learning and habits but where learning is exogenous to the firm. ρ_q falls to 0.73 in this case. This number, while smaller, is still fairly close to the $\rho_q = 0.8$ that we get in our baseline model where learning is endogenized by the firm. Hence, we conclude that our results are not very sensitive to whether learning is internal or external to the firm. What is key is that there is an endogenous mechanism in the

model which can propagate shocks through a real channel with the dominant factor being the fall in future marginal cost due to higher output today.⁹

We next turn to studying the sensitivity of the results for our baseline model with both LBD and habits. Table 3 reports results when we change parameters in our baseline model with both LBD and habits. There are four key parameters of interest in the sense that results are sensitive to them. In Table 3 below we report them in turn. The first margin of interest is the effect of learning. In particular, we have assumed moderate degrees of learning effects. For our baseline case we assumed $\varepsilon = 0.4$ which translates to 32 percent learning, i.e., a doubling of production experience reduces costs by 32 percent. In the empirical literature, estimated learning effects range from 20 percent (see Irwin and Klenow (1994)) to 39 percent (see Benkard (2000)). What is the effect of weaker learning effects? The row “Low LBD” reports results when we lower ε to 0.264 which implies 20 percent learning. Note that we keep all other parameters unchanged relative to the baseline model except ν, ψ and θ which are altered appropriately to target the volatility of investment, the steady state proportion of work time, and the steady state capital-output ratio. As Table 3 makes clear, the primary effects of lower learning are on the persistence of the real exchange rate. In particular, the persistence of the real exchange rate declines to 0.67 from 0.8 in the baseline case. We interpret this result as being suggestive of the quantitative importance of learning effects in understanding the persistence of real variables like the real exchange rate at business cycle frequencies.

The next parameter of interest is b which is the intensity of habits in leisure. Recall that we had assumed $b = 0.8$ which was at the high end of available estimates for b . In the row labeled “LBD plus Low Habits” we report results from our baseline model with $b = 0.5$ holding all other parameters except ν and ψ constant. Row 7 of Table 3 shows that less intensive habit formation in leisure induce less persistence in the real exchange rate which falls to 0.65.

⁹We should note that our model doesn’t allow for entry of new firms. If some of the higher demand generated by a surprise money injection were met through entry of new firms the learning effect would get muted. This then would reduce the degree of persistence generated by the model.

A third parameter that is crucial for our results is the degree of home bias. In the model the degree of home bias is controlled by the parameter a_1 which is the coefficient on home intermediate goods in the final goods technology. In the baseline case we had set $a_1 = 0.94$ to reflect the high home bias in the US data as measured by the share of imports in GDP. To study the importance of home bias, the last row of Table 3 reports results for the case $a_1 = 0.5$. This is the symmetric case in which there is no home bias at all. The results make clear that in the absence of home bias there would be no persistence whatsoever in this model despite the presence of endogenous persistence mechanisms in the form of LBD and habits. In effect, the results under the symmetric case look like the benchmark model in terms of the behavior of the real exchange rate. Intuitively, an unanticipated increase in money supply causes an increase in domestic demand on impact due to preset prices. If this increased demand induces an equal increase in demand for home and foreign intermediates then the production side responds symmetrically in both countries. This implies that the learning and habit effects are symmetric as well which causes the pricing behavior of domestic and foreign firms to be identical. Thus, the domestic and foreign price levels respond symmetrically which, in turn, causes the real exchange rate to revert to its long run equilibrium level in the first period that prices are free to adjust. Note that even though the behavior of the real exchange rate is markedly different from the baseline case, the other moments of the model remain very similar to the baseline case.

A last parameter of interest is the coefficient of risk aversion parameter σ . We use $\sigma = 11$ in order to generate $\sigma_{q/Y}$ approximately consistent with the data. Contrastingly, CKM used $\sigma = 5$. The difference between these two numbers is purely due to the different calibration strategies. We calibrated the adjustment cost parameter ν to target the relative volatility of investment. CKM, on the other hand, picked ν to target the relative volatility of consumption. Our calibration choice implies that the volatility of consumption is too low (relative to output). Hence, the required σ is higher than in CKM. This trade-off between the relative volatility of investment and consumption is evident when comparing rows three and four in Table 2 where we reported moments for CKM. In order to check the sensitivity of our results to this parameter, we followed the CKM strategy

and set $\sigma = 5$ and raised ν enough to match the relative volatility of consumption. The last row of Table 3 reports the results under this alternative parameterization. Relative to our baseline model, this change has a very marginal effect on the persistence of the real exchange rate with ρ_q falling to 0.76 from 0.8 in the baseline case. The volatility of the real exchange rate declines more as $\sigma_{q/Y}$ falls to 4.13 from 5.67 in the baseline case. These numbers are roughly consistent with CKM’s four quarter price stickiness case reported in Table 2. However, despite this, the generated relative real exchange rate volatility is still about 75 percent of the relative volatility in the data (5.5). Clearly, the model (as indeed the CKM model as well) cannot simultaneously match the relative volatilities of both consumption and investment since now the investment volatility generated by the model is too low relative to the data. Overall, we interpret these results as suggesting that our main findings on persistence and volatility are robust to alternative parameterizations of σ .

Given our modeling choices, in particular, the endogenous productivity margin implied by the LBD channel, and the exogenous price rigidity implied by one-period preset prices, we need to address two additional implications of the model. First, the learning mechanism formalized here implies that productivity evolves endogenously in the model. Given that we have calibrated the money shock process so that the output movements in model replicate the data, a logical question is what are the properties of the Solow residuals generated by the model and how do they compare with the Solow residuals measured in the data? The model performs fairly well along this dimension. The standard deviation of the Solow residual relative to output in the model is 0.72 while the first order autocorrelation coefficient of the Solow residual is 0.96. Since both these numbers are close to those reported in the data, we conclude that the model is generating sensible paths for capital and hours.

Second, while our model has highlighted an environment with minimal price rigidity (one quarter), a key criticism of models with exogenous price rigidity and local currency pricing (LCP) is that these models predict that the correlation between the nominal exchange rate and the terms of trade (defined as the ratio of import to export prices) should be negative, i.e., when the currency depreciates, the terms of trade facing the country should improve. Pointing this out, Obstfeld

and Rogoff (2000) showed evidence that the contemporaneous correlation between changes in the exchange rate and changes in the terms of trade was positive for most countries in their sample. Obstfeld and Rogoff concluded from their work that models with hard-wired price rigidity and LCP like CKM (2002) were problematic. While our model has both one period pre-set prices and LCP built into them, we do have other mechanisms in the model (LBD and habits) which induce endogenous sluggish price adjustment. Hence, the model might be able to escape the Obstfeld-Rogoff criticism. Indeed, the contemporaneous correlation between innovations in the nominal exchange rate and the terms of trade is positive (0.24).

The key to this result is the learning mechanism. Consider a positive shock to the rate of growth of money supply at home. The nominal exchange rate depreciates on impact but the terms of trade (p_f/ep_h^*) falls since p_f and p_h^* are both fixed. Hence, the impact effect goes exactly as in the Obstfeld-Rogoff criticism. However, next period the nominal exchange rate depreciates some more as the rate of growth of money remains high. On the other hand, the higher production in the impact period implies that both domestic and foreign intermediate goods producers find their productivities to be higher next period. This reduces both their marginal costs. Hence, they both raise their prices by less than the full proportion of the shock. However, this productivity effect is much stronger for the home producer than for the foreign producer. Since the money shock increases the demand for final goods at home, the home bias in the production technology for intermediate goods implies that the demand for home intermediate goods rises more than the demand for foreign goods. Hence, the learning effect on organizational capital is higher at home than abroad. This induces the domestic intermediate producer to raise prices less than the foreign supplier. This differential effect induces the terms of trade to deteriorate over time as the nominal exchange rate depreciates.

5 Conclusions

Traditional explanations for the high volatility and persistence of real exchange rates in the data have been based on monetary shocks and nominal price rigidities. However, recent work on

standard dynamic general equilibrium models with sticky prices has shown that this traditional explanation requires prices to be rigid for upwards of four quarters (see Chari, Kehoe and McGrattan (2002)). This appears to be too high relative to the data (e.g., Bils and Klenow (2004)). In this paper we have augmented the standard model with learning-by-doing at the level of the firm and habit formation in leisure. These two margins introduce an endogenous propagation mechanism into the standard model. As a result, even with one period pre-set prices, a monetary expansion has long-lived effects. The increase in production in the period of the shock causes learning which raises future productivity and output. Hence, firms raise prices by less than the full proportion of the money shock in the following period since the marginal cost of production is low and thus produce more than the long run equilibrium level. This channel is complemented by habits in leisure which induce labor supply to adjust gradually. As a result, the dynamics of output and the real exchange rate both show persistent deviations from steady state. Under our baseline calibration, the model generates a first-order autoregression coefficient of 0.8 and a standard deviation (relative to output) of 5.6 for the real exchange rate even though prices are fixed for only one quarter and monetary shocks are the only source of variability. We view these results as both providing an explanation for real exchange rate behavior as well as being supportive of the basic mechanism formalized in the paper.

More generally, the mechanism formalized in the model introduces an endogenous propagation channel which can transmit and propagate disturbances in real variables over time. Hence, even though in this paper we have studied business cycle dynamics that are driven by monetary shocks alone, we believe that the model would work well in alternative environments where business cycle dynamics are driven by either technology shocks or both technology shocks and monetary shocks. Moreover, since prices are pre-set for just one quarter, the model relies essentially on its internal real transmission mechanism to generate persistence rather than exogenous price stickiness. Hence, we believe that the quantitative results will go through under richer or more realistic monetary policy rules like the Taylor rule rather than the simple money growth process that we have studied here since the sole role of a monetary shock here is to cause an initial disturbance in output.

We have chosen to ignore some important and relevant margins in this paper. Thus, we have assumed that producers price to market (PTM) and price their goods in terms of the currency of the buyer (local currency pricing). We do not explain why producers choose to behave this way. Interesting work on this topic can be found in Burstein, Eichenbaum and Rebelo (2005) and Corsetti, Dedola and Leduc (2006) who study the role of distribution costs and non-traded goods in understanding the behavior of real exchange rates.

Another issue relates to the model's predicted pricing behavior at the firm level. In particular, under the environment of the model, the representative firm changes prices relatively smoothly and in every period in response to a monetary shock. This is at odds with the available evidence on firm-level pricing behavior which suggests that firms adjust prices infrequently and when they do they adjust prices discretely. As an illustration, Golosov and Lucas (2007) say that conditional on changing prices, on average, firms increase them by 9.5 %. However, it is not clear if these changes are driven solely by changes to monetary policy. Thus, Golosov and Lucas try to match the firm-level facts by building a model with both idiosyncratic shocks and aggregate monetary shocks. If the observed large changes in prices at the firm level are driven by large idiosyncratic shocks, then an appropriately modified version of our model to include firm level shocks may be able to account for it. However, given that the goal of the model is to explain aggregate responses to monetary shocks rather than individual price data, this kind of extension would take us well beyond the scope of the paper. As is often the case when comparing micro and aggregate data, individual price changes display a lot of lumpiness, but adjustments in the aggregate price level are quite smooth. Our model delivers this feature.

References

- [1] Arrow, K.J., 1962. The Economic Implications of Learning-by-Doing. *Review of Economic Studies* 29, 155-173.
- [2] Atkeson, A., Kehoe, P.J., 2005. Modeling and Measuring Organizational Capital. *Journal of Political Economy* 113, 1026-1053.
- [3] Backus, D., Kehoe, P.J., Kydland, F., 1994. The Dynamics of the Trade Balance and the Terms of Trade: The J-Curve. *American Economic Review* 84, 84-103.
- [4] Bahk, B-H., Gort, M., 1993. Decomposing Learning By Doing in New Plants. *Journal of Political Economy*, 101(4), 561-583.
- [5] Benkard, L., 2000. Learning and Forgetting: The Dynamics of Aircraft Production. *American Economic Review* 90, 1034-1054.
- [6] Bills, M., Klenow, P., 2004. Some Evidence on the Importance of Sticky Prices. *Journal of Political Economy* 112, 947-985.
- [7] Bergin, P., Feenstra, R., 2001. Pricing-to-Market, Staggered Contracts, and Real Exchange Rate Persistence. *Journal of International Economics* 54, 333-359.
- [8] Betts, C., Devereaux, M., 2000. Exchange Rate Dynamics in a Model of Pricing-to-Market. *Journal of International Economics* 50, 215-244.
- [9] Bouakez, H., 2005. Nominal Rigidity, Desired Markup Variations and Real Exchange Rate Persistence. *Journal of International Economics* 66, 49-74.
- [10] Bouakez, H., Kano, T., 2006. Learning-by-Doing or Habits. *Review of Economic Dynamics* 9, 508-524.
- [11] Braun, R. A., Evans, C.L., 1998. Seasonal Solow residuals and Christmas: a case for labor hoarding and increasing returns..*Journal of Money, Credit & Banking* 30, 306-330.
- [12] Burstein, A., Eichenbaum, M., Rebelo, S., 2005. Large Devaluations and the Real Exchange Rate. *Journal of Political Economy* 113, 742-784.
- [13] Chang, Y., Gomes, J., Schorfheide, F., 2002. Learning-by-doing as a Propagation Mechanism.

- American Economic Review 92, 1498-1520.
- [14] Chari, V.V., Kehoe, P., McGrattan, E., 2000. Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem? *Econometrica* 68, 1151-1179.
 - [15] Chari, V.V., Kehoe, P., McGrattan, E., 2002. Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates. *Review of Economic Studies* 69, 533-563.
 - [16] Cooper, R., Johri, A., 2002. Learning-by-doing and Aggregate Fluctuations. *Journal of Monetary Economics* 49, (8) 1539-1566.
 - [17] Corsetti, G., Dedola, L., Leduc, S., 2006. DSGE Models with High Exchange Rate Volatility and Low Passthrough. *International Finance Discussion Papers of the Federal Reserve Board*, no. 845.
 - [18] Dornbusch, R., 1976. Expectations and Exchange Rate Dynamics. *Journal of Political Economy* 84, 1161-1176.
 - [19] Dornbusch, R., 1987. Exchange Rates and Prices. *American Economic Review* 77, 93-106.
 - [20] Eichenbaum, M., Hansen, L., Singleton, K., 1988. A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice under Uncertainty. *Quarterly Journal of Economics* 103, 51-78.
 - [21] Golosov, M, Lucas Jr., R.E., 2007. Menu Costs and Phillip Curves. *Journal of Political Economy* 115, 171-199.
 - [22] Hotz, V., Kydland, F., Sedlacek, G., 1988. Intertemporal Preferences and Labor Supply. *Econometrica* 56, 335-360.
 - [23] Irwin, D., Klenow, P., 1994. Learning-by-doing Spillovers in the Semiconductor Industry. *Journal of Political Economy* 102, 1200-1227.
 - [24] Jarmin, R.S., 1994. Learning by Doing and Competition in the Early Rayon Industry. *RAND Journal of Economics* 25, 441-454.
 - [25] King, R.G., Watson, M.W., 2002. System reduction and solution algorithms for singular linear difference systems under rational expectations. *Computational Economics* 20, 57-86.
 - [26] Kocherlakota, N., Pistaferri, L., 2006. Household Heterogeneity and Real Exchange Rates.

- forthcoming *Economic Journal*.
- [27] Kydland, F., Prescott, E., 1982. Time to Build and Aggregate Fluctuations. *Econometrica* 50, 1345-1371.
 - [28] Lev, B., Radharkrishnan, S. (2003) The Measurement of Firm-Specific Organizational Capital. NBER Working Paper # 9581.
 - [29] Obstfeld, M., Rogoff, K., 1995. Exchange Rate Dynamics Redux. *Journal of Political Economy* 103, 624-660.
 - [30] Obstfeld, M., Rogoff, K., 2000. New Directions for Stochastic Open Economy Models. *Journal of International Economics* 50, 117-154.
 - [31] Prescott, E., Visscher, M., 1980. Organization Capital. *Journal of Political Economy* 88, 446-61.
 - [32] Ravn, M., Mazzenga, E., 2004. International Business Cycles: the quantitative role of transportation costs. *Journal of International Money and Finance* 23, 645-671.
 - [33] Rosen, S., 1972. Learning by Experience as Joint Production. *Quarterly Journal of Economics* 86, 366-382.
 - [34] Thornton, R., Thompson, P., 2001. Learning from Experience and Learning from Others: An Exploration of Learning and Spillovers in Wartime Shipbuilding. *American Economic Review* 91, 1350-68.
 - [35] Yun, T., 1996. Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles. *Journal of Monetary Economics* 37, 345-370.

TABLE 1. PARAMETERS FOR BASELINE CALIBRATION

Parameter	Value	Comment
ω	0.94	Preference parameter
φ	0.39	Preference parameter
ψ	Calibrated	Preference parameter
σ	11	Intertemporal risk aversion parameter
ξ	σ	Controls elasticity of labor supply
b	0.8	Habit intensity
β	0.984	Discount factor
ρ	1/3	Controls the elasticity of substitution between home and foreign goods
θ	Calibrated	Controls elasticity of substitution between home goods
a_1	0.94	Home goods production share parameter
a_2	0.06	Foreign goods production share parameter
α	0.6	Labor share of output
ϕ	0.4	Capital share of output
δ	0.021	Depreciation rate
ν	Calibrated	Adjustment cost parameter
γ	0.5	Coefficient on H_t in learning function
η	1	Coefficient on Y_t in learning function
ε	0.4	Output share of organizational capital

TABLE 2: PROPERTIES OF THE MODELS

	$\sigma_{C/Y}$	$\sigma_{I/Y}$	$\sigma_{N/Y}$	$\sigma_{q/Y}$	$\sigma_{e/Y}$	ρ_q	ρ_e	$\rho_{q,e}$
US Data	0.95	2.66	0.78	5.50	6.29	0.94	0.95	0.99
Benchmark	0.55	2.66	1.65	5.65	7.48	-0.01	0.66	0.92
LBD + Habits	0.55	2.66	1.22	5.67	5.11	0.8	0.76	0.97
CKM 4 quarters	0.83	1.58	2.44	4.24	6.93	0.71	0.89	0.49
LBD only	0.56	2.66	1.45	5.18	6.20	0.48	0.73	0.92
Habits only	.55	2.66	1.66	5.35	6.40	.45	.67	0.97
Exog. LBD+Habits	0.55	2.66	1.30	5.64	5.48	0.73	0.72	0.97

TABLE 3: SENSITIVITY OF RESULTS

	$\sigma_{C/Y}$	$\sigma_{I/Y}$	$\sigma_{N/Y}$	$\sigma_{q/Y}$	$\sigma_{e/Y}$	ρ_q	ρ_e	$\rho_{q,e}$
US Data	0.95	2.66	0.78	5.50	6.29	0.94	0.95	0.99
LBD + Habits	0.55	2.66	1.22	5.67	5.11	0.8	0.76	0.97
Low LBD + Habits	0.55	2.66	1.33	5.53	5.72	0.67	0.71	0.99
LBD + Low Habits	0.55	2.66	1.33	5.36	5.81	0.65	0.74	0.98
No Home Bias	0.55	2.66	1.18	5.27	6.96	0	0.65	0.76
$\sigma = 5$	0.95	1.22	1.16	4.13	3.78	0.76	0.72	0.98